New Coordinate Systems for Axisymmetric Black Hole Collisions

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We describe a numerical grid generating procedure to construct new classes of orthogonal coordinate systems that are specially adapted to binary black hole spacetimes. The new coordinates offer an alternative approach to the conventional Čadež coordinates, in addition to providing a potentially more stable and flexible platform to extend previous calculations of binary black hole collisions.

1 Introduction

The two–body problem is a milestone in numerical relativity that has not yet been attained. Even the relatively simple case of the head–on collision of two equal mass, non-rotating black holes remains uncertain for black holes that are initially separated by large distances. By exploiting the 2–dimensional nature of the problem, the obvious first choice of coordinates in which to solve the Einstein equations for the axisymmetric collision of two black holes are the cylindrical ones. However, these coordinates suffer from a code–crashing axis instability that can be suppressed (somewhat) by adding a shift vector to maintain a diagonal metric, but at the expense of introducing steep shear features in the solutions that cannot be maintained in a stable manner. An alternative approach, developed more than 20 years ago, is to perform the evolutions using curvilinear Čadež coordinates[1]. However, these evolutions are also highly unstable due to the saddle–point singularity in the coordinate system. Two methods that have proven to be more successful invoke elements of both the cylindrical and Čadež coordinates[2,3]. These evolutions run well when the black holes are restricted to separations less than about $13M$, where M is the single black hole mass. However, even the most accurate of these methods[3] is only a partial success when applied to the Misner initial data since the code cannot evolve data with large separation parameters.

In this paper, we present an alternative approach to the axisymmetric binary black hole collision by constructing two new classes of orthogonal body–fitting coordinate systems, specially adapted to the 2–body problem.

2 A New Approach

While saddle–point singularities are unavoidable in generating the appropriate body–fitting coordinates for two disconnected domains, it is reasonable to suppose that problems associated with these singularities can be minimized if the saddle–points were moved from the origin (as in the Čadež grid, Fig. 1) to either the "south pole" (class I coordinates, Fig. 2), or to the "north pole" (class II coordinates, Fig. 3) of the top hole. By setting the lapse to zero on the black hole throats, the system is

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prevented from evolving in regions near the saddle–points and this contributes to the stability of evolutions in the rest of the spacetime.
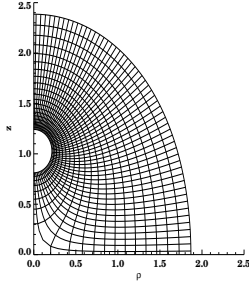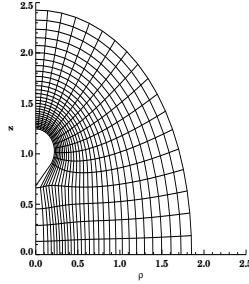
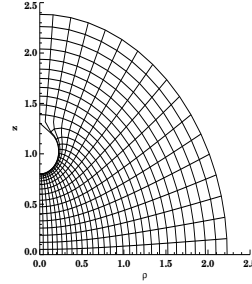

Fig. 1: Čadež grid. Fig. 2: Class I grid. Fig. 3: Class II grid.

Our grid generation procedure is based on a semi–analytic approach: In the class I case, we construct an analytic *radial* coordinate specially designed to enforce the proper boundary conditions. The companion angular coordinate is computed numerically by integrating a set of ODEs that satisfy the orthogonality conditions for each coordinate line. In the class II case, we construct an analytic *angular* coordinate and evaluate the companion radial coordinate numerically. For example, the class I radial coordinate is constructed from three separate distance measures

$$r_1 = \sqrt{\rho^2 + (z - z_0)^2} - a, \tag{1}$$

$$r_2 = \sqrt{\rho^2 + (z + z_0)^2} - a, \tag{2}$$

$$r_3 = \left(\sqrt{\rho^2 + (z - z_0 + a)^2} + \sqrt{\rho^2 + (z + z_0 - a)^2} - 2z + 2a \right)/2, \tag{3}$$

where a is the radius of the black holes, and $z = \pm z_0$ are the locations of the black hole centers. r_1 and r_2 represent distances from the two throat surfaces to a point (ρ, z) in conformal space. The third radius is an "elliptic distance" from the central line segment connecting the two holes. Each of the different radii are zero on different parts of the inner spectacle–shaped boundary composed of the two throats and the line segment connecting them. When combined appropriately, ie. $r = 3r_1 r_2 r_3 / (r_1 r_2 + r_2 r_3 + r_1 r_3)$, they form a coordinate that is zero along the entire inner boundary and becomes spherical at large distances.

3 Evolutions

To date, we have performed several evolutions of black hole collisions with the new class I coordinates. Our preliminary results indicate that the extracted waveforms are less sensitive to the grid and coordinate patch parameters (which we continue to use over the first few radial zones) than in the Čadež evolutions. The evolutions are also able to run for longer times with large initial separations of the black holes. Agreement between the different codes is generally good for moderately separated black holes ($\mu \leq 2.5$ in the Misner parameter). A comparison of the dominant $\ell = 2$ waveforms between the Čadež and class I grids is presented in Fig. 4 for the $\mu = 2.2$

case. We continue to investigate the further separated black hole cases, as well as the class II system, and will report the results in a more comprehensive paper[4].

With the new class I grids, it is possible to track the event horizon and locus of null generators[5] more accurately, since their geometries conform more closely to the new coordinates than the Čadež ones (which are singular at the critical merger point). The embedding of the event horizon is shown in Fig. 5 for $\mu = 2.2$. We note that the spacing between the embedded horizons is not arbitrary (as it was in the Čadež case), due to the shapes of the horizon and locus in the class I coordinate system.

4   Conclusions

We have developed two new classes of coordinate systems and demonstrated the applicability of the class I type in actual numerical evolutions of colliding black holes. The new coordinates appear capable to improve on existing axisymmetric calculations of two black hole collisions. The evolutions are more stable, the waveforms less sensitive to patch parameters, and the embedding of the horizon can be determined more precisely and with greater ease. We continue to develop these methods for class I and II grids, and for evolutions of black holes with large initial separations.
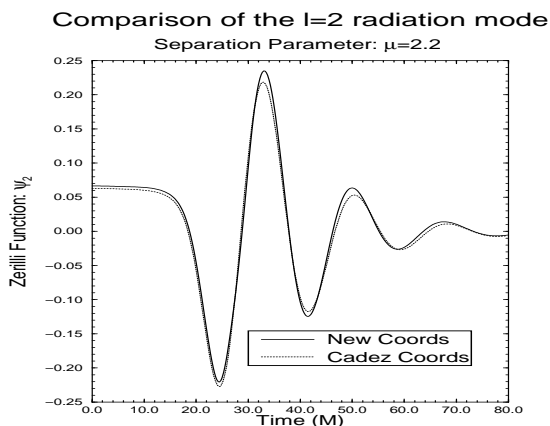

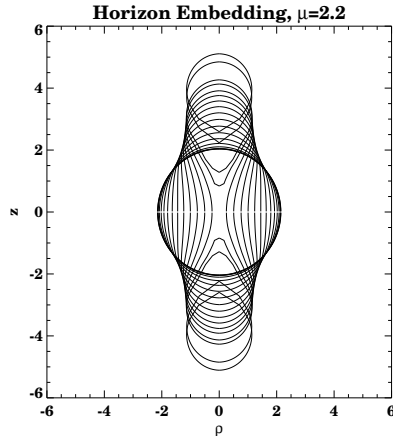Fig. 4: $\ell = 2$ waveform comparison between the class I and Čadež grids.

Fig. 5: Embedding showing the merger of the class I event horizons.

References

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